DOCUMENT RESUME

ED 029 792 SE 006 733

By-Beckmann, Milton W.

An Appraisal of Recent Developments in the Curriculum and Teaching of Mathematics at the Secondary Level.

Great Plains School District Organization Project, Lincoln, Neb.

Pub Date Apr 68

Note-31p.

EDRS Price MF-$0.25 HC-$1.65

Descriptors-Algebra, Comparative Analysis, *Curriculum, *Curriculum Development, Geometry, *Instruction, *Mathematics Education, Number Concepts, *Secondary School Mathematics, Teacher Education

This paper appraises recent developments in the curriculum and teaching of mathematics in the secondary schools and analyzes mathematics education in the United States with particular emphasis on Nebraska. The first section focuses upon characteristics that distinguish the modern secondary school mathematics curriculum from the traditional. The following sections emphasize the understanding and the structure of mathematics. Ability groupings and the various types of programs which have resulted from it are also discussed. Further analyses deal with a college preparatory sequence for Grades 9-12, new mathematics programs to attract and to motivate students, and new techniques in mathematics at the secondary level. Suggestions for implementing new developments are also provided. (RP)

AN APPRAISAL OF RECENT DEVELOPMENTS IN THE CURRICULUM AND TEACHING OF MATHEMATICS AT THE SECONDARY LEVEL

by

Milton W. Beckmann

Supervisor of Mathematics and
Professor of Secondary Education
University of Nebraska
Lincoln, Nebraska

April, 1968

The Great Plains School District Organization Project
Iowa, Missouri, Nebraska, South Dakota
Ralph D. Purdy, Project Director
411 South 13th Street
Lincoln, Nebraska
68508

FOREWORD

The impact of scientific, technological, social, and economic change on the American way of life necessitates a re-examination of the educational system. These changes modify established needs and create new needs to be met by the public school system. Instructional programs and supporting services must be developed to meet these needs.

The primary purposes of school district organization are to make possible: (1) the desired quality or excellence of the programs and services; (2) the efficiency of the organization for providing the programs and services; and (3) the economy of operation, or the maximum returns received for the tax dollar invested in education.

A revolution has been occurring in the field of mathematics. The "old" math is being replaced by the "new" math. At the same time mathematics, old or new, continues to be one of the basic essentials for science, vocational programs, business and industry, and for every day personal and business management. Dr. Milton W. Beckmann, Supervisor of Mathematics, University of Nebraska, was invited to prepare a paper that would briefly identify the transition that is taking place in the broad field of mathematics, and to indicate significant relationships in quality, offering, or related factors to school district organization. This paper is his report to the Project Staff.

The value of this paper rests upon its utilization by those with advisory and/or decision making responsibilities about the educational structure in each state. It represents a beginning point for further study and evaluation, and for establishing criteria upon which guidelines can be developed for effective and constructive school district organization.

Respectfully submitted,

Ralph D. Purdy, Director
Great Plains School District
Organization Project

April, 1968

AN APPRAISAL OF RECENT DEVELOPMENTS IN THE CURRICULUM AND TEACHING OF MATHEMATICS AT THE SECONDARY LEVEL

The purpose of this paper is to present an appraisal of recent developments in the curriculum and teaching of mathematics in the secondary schools. The paper presents a critical analysis of the situation in the United States and in Nebraska regarding mathematics education.

CHARACTERISTICS THAT DISTINGUISH THE MODERN SECONDARY SCHOOL MATHEMATICS CURRICULUM FROM THE TRADITIONAL

Attached to this paper is a list[1] of characteristics that distinguish the modern secondary school mathematics curriculum from the traditional mathematics curriculum as found in the study on modern mathematics, which was conducted by Ronald O. Massie,[2] past Supervisor of Mathematics at University High School. Massie developed a test which was given to prospective and experienced teachers in eight states to determine how much modern mathematics they knew. Massie, after reading the literature, evaluating textbooks, and the like, listed a number of characteristics that distinguish the modern secondary school mathematics curriculum from the traditional mathematics curriculum. He sent the list to twelve leaders in mathematics education including, Begle[3], Beberman[4], Johnson[5], Wells[6], Fehr[7], and Bezuzska[8]. Massie created a procedure so these leaders could indicate the extent of their agreement with each characteristic as being a distinguishing characteristic of modern mathematics. These characteristics are common to "modern mathematics" courses at all levels.

"Inequalities treated along with equations" received the highest rating. (See number 3 of the list found at the top of page 27.) "Structure emphasized" received the second highest rating. (See number 1 of the list found on page 26.) The third ranking characteristic is number 1 found at the bottom of page 27, entitled "Emphasis in trigonometry changed." Ranking fourth were two items. One of these items, "Function concept introduced", is found at the top of page 27, and "Understanding emphasized", is number 3, page 26. In fifth place, three characteristics were listed.

---

[1] Massie, Ronald O., The Construction and Use of a Test to Evaluate Teachers Preparation in Modern Mathematics, Doctor's Dissertation, University of Nebraska, Lincoln, 1967.

[2] Massie, Ronald O., Assistant Professor of Mathematics, University of Toledo.

[3] Begle, E. G., Director of the School Mathematics Study Group.

[4] Beberman, Max, Director of the University of Illinois Committee on School Mathematics.

[5] Johnson, Donovan, President of the National Council of Teachers of Mathematics.

[6] Wells, David, Director of Mathematics Education, Oakland County Schools in Michigan.

[7] Fehr, Howard, Past President of the National Council of Teachers of Mathematics.

[8] Bezuzska, Stanley, Director of the Boston College Mathematics Series and Chairman of the Mathematics Department, Boston College.

Space will not permit a complete discussion of all characteristics, but let us look at some of them.

UNDERSTANDING EMPHASIZED

Let us discuss the characteristic, Understanding Emphasized. This characteristic is number 3, on page 26.

According to some curriculum reformers, a good program of mathematics depends as much on the teacher's approach as on what is being taught. Some educators have pointed out that little is gained if the new mathematics is taught in an uninspired way.

With this in mind, many of the leaders in mathematics education have made a strong appeal for the use of an approach to teaching called the discovery method. The philosophy and methodology of the discovery approach are subtle and involved. A few of its major premises are pointed out by Deans as follows:

1. We almost never tell the student what to do, nor how to do it. Instead, we ask questions. The student learns by thinking through the questions himself.

2. We try to get the student thinking about the basic concepts as early as possible. We avoid the use of extensive vocabulary.

3. We usually conduct classes with a maximum of student participation. Every effort is made to get the students thinking and talking. We do not want them merely listening and accepting.

4. In our experience, nearly every student answer has some merit. We respond to every student answer as we believe scientists should -- with respect.

5. Our real purpose is to explore mathematics and to get students thinking on their own about the basic concepts of mathematics. We want them to enjoy it and to be motivated to learn more.[9]

The discovery approach, according to its proponents tends to make a pupil confident and capable in mathematics. It helps him enjoy the subject. If it does these things, then it is truly wonderful. However, everything is not rosy. Teachers, trying to use the methods of dis very, are reporting problems. Some of these problems are listed by Hawthorne:

1. Teachers need a wide background in mathematics as well as great confidence and patience.

2. It is very difficult to use this approach with students who have had no previous experience with it.

3. The gains in confidence, liking for the subject, and understanding, are difficult to measure.

---

[9]Deans, Edwina, Elementary School Mathematics, New Directions, U.S. Department of Health, Education, and Welfare, Washington, D. C., 1963, page 65.

4. Children of low ability occasionally seem disturbed by the discovery approach.

5. In some classes just a few of the children are discovering while the majority listen.[10]

Even with these disadvantages, I think the approach has much merit. I taught a class of seventh graders for a year using the materials developed by the School Mathematics Study Group.[11] I want to show you why I got excited about the program.

Gauss' Addition of Consecutive Numbers

Karl Gauss was one of the most outstanding mathematicians of all time. The story is told that when Gauss was in the fifth grade his teacher asked all members of his class to add the numbers from one through 100. I imagine it was on a Friday afternoon when the teacher was somewhat weary. This exercise might have served to keep the students busy for the remainder of the afternoon. Gauss had the answer within a few minutes. He discovered a new technique which is shown below.

The Development of Gauss' Formula

(for finding the sum of an arithmetical progression)

The sum of the numbers from 1 to 100:

$$\begin{array}{r} 1 + 2 + 3 + 4 \ldots\ldots + 98 + 99 + 100 \\ 100 + 99 + 98 + 97 \ldots\ldots + 3 + 2 + 1 \\ \hline 101 + 101 + 101 + 101 \ldots\ldots + 101 + 101 + 101 \end{array}$$

$$101 + 101 + 101 + 101 \ \ldots + 101 = 101\ (1 + 1 + \ldots + 1)$$
$$= 101 \times 100 = 10{,}100$$

But this is using each number twice. For example, we added 1 + 100 and 100 + 1, 2 + 99, and 99 + 2...therefore, we must divide our sum by 2:

$$\frac{\text{Sum}}{2} = \frac{10{,}100}{2} = 5{,}050$$

Note: $\text{Sum} = \dfrac{101 \text{ X } 100}{2} = \dfrac{(1 + 100)\ (100)}{2} = \dfrac{(A + L)\ N}{2}$

where A is the first number or term, L is the last number or term and N is the number of terms of the progression.

[10] Hawthorne, Frank S., Chief, Bureau of Mathematics Education, New York State Education Department, Elementary School Mathematics, New York State School Boards Association, Inc., Vol. 8, No. 6, December, 1966.

[11] Begle, E. G., Director, School Mathematics Study Group, School of Education, Stanford University, Stanford, California, 94305.

One can find the sum of any series of consecutive numbers by adding the first number and the last number, then multiplying by the number of numbers and dividing by two. Therefore:[12]

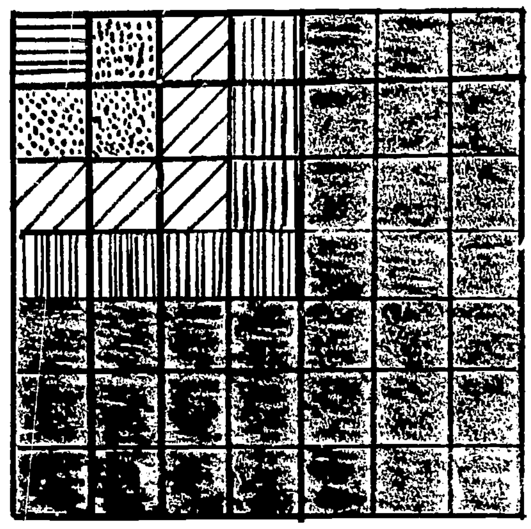
$$S = \frac{(A + L)\ N}{2}$$

Addition of Odd Numbers

Numbers Related to Squares

$$\begin{array}{r}
1 = 1 \\
1 + 3 = 4 \\
1 + 3 + 5 = 9 \\
1 + 3 + 5 + 7 = 16 \\
1 + 3 + 5 + 7 + 9 = 25 \\
1 + 3 + 5 + 7 + 9 + 11 = 36 \\
1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 \\
\vdots \\
1 + 3 + \ldots\ldots\ 31 + \ldots\ldots\ 58 + 59 + 61 = 961
\end{array}$$

Geometric Pattern of Squares



The reader will observe that in each row of numbers an additional number is added. Each time the geometrical square is increased by an odd number of squares. The number of small squares added each time is equal to the "number" added to the next row. The students are to "discover" that the sum of the odd numbers is equal to the square of a number. The number squared is the middle "number" of the series or the number of numbers added.

Goldberg's Formula

Below you will observe that we are finding the sum of a geometrical series. A young Jewish boy named Melvin Goldberg who had just completed the sixth grade, discovered the formula.

EXAMPLE:

$1 + 3 + 9 + 27 \ldots = ?$

$$S = \left[\left(\frac{1}{R-1}\right) L\right]^{*} - 1 + L$$

(Goldberg's Formula)

$$S = \left[\left(\frac{1}{3-1}\right) 27\right] - 1 + 27$$

$$S = \left[\frac{27}{2}\right]^{*} - 1 + 27 = 40$$

$$S = 14 - 1 + 27 = 40$$

*This fraction is expressed as the next integer: R is the common ratio, L is the last term.

[12] Gauss' formula for finding the sum of any series of consecutive numbers.

Multiplication of Rational Numbers

	-3	-2	-1	0	1	2	3
+3				0	3	6	9
+2				0	2	4	6
+1				0	1	2	3
0	0	0	0	0	0	0	0
-1				0			
-2				0			
-3				0			

Figure A

	-3	-2	-1	0	1	2	3
+3	-9	-6	-3	0			
+2	-6	-4	-2	0			
+1	-3	-2	-1	0			
0	0	0	0	0	0	0	0
-1				0	-1	-2	-3
-2				0	-2	-4	-6
-3				0	-3	-6	-9

Figure B

	-3	-2	-1	0	1	2	3
+3				0			
+2				0			
+1				0			
0	0	0	0	0	0	0	0
-1	3	2	1	0			
-2	6	4	2	0			
-3	9	6	3	0			

Figure C

	-3	-2	-1	0	1	2	3
+3	-9	-6	-3	0	3	6	9
+2	-6	-4	-2	0	2	4	6
+1	-3	-2	-1	0	1	2	3
0	0	0	0	0	0	0	0
-1	3	2	1	0	-1	-2	-3
-2	6	4	2	0	-2	-4	-6
-3	9	6	3	0	-3	-6	-9

Figure D

By studying the figures above can you discover the sign to be used when multiplying a negative number by a positive number? When multiplying a negative number by a negative number? It is plausible to reason that a plus number times a negative equals a negative, and a negative number times a negative number is plus. This is not proof, but one can discover the answer through patterns.

ANOTHER CHARACTERIZING FEATURE OF MODERN MATHEMATICS IS THE "EMPHASIS ON THE STRUCTURE OF MATHEMATICS."

We have discussed "discovery" as a characterizing feature of modern mathematics. Would you not agree that students making these discoveries would be fascinated? Another characterizing feature of the new mathematics is the emphasis being placed on the structure of mathematics. The structure of any mathematical system is determined by the set of elements and the properties characteristic of these elements. Fields, groups, rings, integral domains are systems which are characterized by their structure.

Let us take a look at the squaring of whole numbers ending in 5.

$25 = 10 \cdot 2 + 5$

$25 = 10 \cdot 2 + 5$ OR $(10 \cdot 2 + 5)(10 \cdot 2 + 5)$

$(10 \cdot 2 + 5)(10 \cdot 2 + 5)$	$= 10\cdot 2(10\cdot 2+5) + 5(10\cdot 2+5)$	Distributive Property
	$= (10\cdot 2\cdot 10\cdot 2+10\cdot 2\cdot 5) + (5\cdot 10\cdot 2+5\cdot 5)$	Distributive Property
	$= 10\cdot 10\cdot 2\cdot 2+10\cdot 2\cdot 5+5\cdot 2\cdot 10+5\cdot 5$	Commutative Property
	$= 10\cdot 10\cdot 2\cdot 2+10\cdot 10+10\cdot 10+5\cdot 5$	Multiply
	$= 10\cdot 10\cdot (2\cdot 2+1+1) + 25$	Distributive Property
	$= 10\cdot 10\cdot (2\cdot 2+2) + 25$	Add
	$= 10\cdot 10\cdot 2\cdot (2+1) + 25$	Distributive Property
	$= 10\cdot 10\cdot 2\cdot 3 + 25$	Add
	$= 600 + 25$	Multiply
	$= 625$	Add

When multiplying any two-digit number ending in five, one observes that the number always ends in 25. Furthermore, one multiplies the number in the ten's digit by the next higher digit, and then multiplies these numbers by 10 X 10. We have discovered a pattern. Does this pattern hold for all cases? Let us generalize. We ask the students for a general form of all two-digit numbers ending in five. See the generalization below:

$10X + 5$ is the number

$(10X+5)(10X+5)$	$= 10X(10X+5) + 5(10X+5)$	Distributive Property
	$= 10\cdot 10X^2+50X+50X + 25$	Distributive Property
	$= 10\cdot 10X^2+100X + 25$	Addition
	$= 10\cdot 10(X^2 + X) + 25$	Distributive Property
	$= 10\cdot 10\cdot X(X+1) + 25$	Distributive Property

Would you agree that emphasizing structure in mathematics to a youngster would be better than memorizing rules? You have seen illustrations of structure in arithmetic and algebra. Next a few words about structure in geometry.

Different Assumptions Result in Different Interpretations

A painting found on the cover of the September, 1964, issue of the Scientific American[13] poetically represents the theme of the issue mathematics in the modern world.

The painting symbolizes that aspect of mathematics which makes outrageous new assumptions to erect new mathematical systems. What do you see through the closed half of the window? One sees a sunlit sea and sky. What does the open half of the window show you? The open half shows that behind the window is nothingness. What does one then assume? One assumes that the scene is painted on the glass. But wait! Look at the top of the open half of the window. What do you see? At the top of the open half of the window one can see the window frame through the glass.

To understand that the rules of the mathematical game are often changed in a way that has nothing to do with so-called physical intuition, let us remember that the geometry we learned in high school was systematized once and for all by Euclid. One of the fundamental hypotheses of Euclid's plane geometry is that one and only one line may be drawn through a point outside a line and parallel to it. This hypothesis seems to be substantiated by our own physical intuition or biological understanding of the world around us, and it is certainly most useful in our daily life. But entirely different hypotheses are all identically acceptable from a theoretical viewpoint, and some of the less obvious ones are just as practically important as the hypotheses of Euclid. Our interpretation of the painting depends upon our basic assumption.

We have illustrated the characteristics of discovery and structure. Space does not permit the discussion of other characteristics of modern mathematics. I do want to mention them -- coordinate geometry, inequalities, and set language. These are new mathematical topics not used in traditional mathematics. This requires that more mathematics be taught because much of the old mathematics needs to remain.

ABILITY GROUPING

In recent years, the general trend toward ability grouping has caused the secondary school mathematics curriculum to fall into general patterns similar to the following:

HONORS PROGRAMS

1. Accelerated Program for Honor Students:

This plan leads either to the equivalent of a college mathematics course in the senior year with the advanced placement examinations, or an honors type of senior mathematics. This is usually accomplished by offering elementary algebra in grade 8 and each of the sequential courses one year earlier than usually given.

---

[13] The painting on the cover is entitled, "Mathematics in the Modern World."

In cities where the grade level pattern is 8-4, some 8th grade pupils go to neighboring senior high schools for their algebra class. Similarly, some high school seniors (for example, in Chicago, Kansas City, Miami, and Pittsburg) take college courses in nearby colleges or junior colleges for advanced placement work.[14]

2. University High School Program:

Two years ago at University High School we offered our seniors the same course that the University of Nebraska offers most freshmen. We had 19 students enrolled. After completing the course, seventeen took the University examination given for students in Mathematics 14, the beginning course in mathematics at the University. Fifteen of our students received four hours of credit at the University. We used the same text as the University. Students in high school no longer can receive University credit for Mathematics 14 by examination.

3. The Pittsburgh Scholars Program

The Pittsburg Scholars Program, a rigorous 5-year program of study for grades 8 through 12, is now available to the top 20 percent of the pupils in the Pittsburg Public Schools. Sequences of study include a 5-year program in mathematics which, for some students, leads to calculus in the fifth year.[15]

4. Tolly's Study

Tolly[16] made a study of the Lincoln Public Schools accelerated mathematics program. Most of the accelerated students did exceedingly well at the University of Nebraska, both in academic work and participation in activities.

Supervisors in a recent Ohio bulletin on Acceleration and the Gifted reported that:

> Statistical evidence seems to indicate the academically talented pupils who were accelerated performed as well (except in solid geometry) as any pupils with whom they were compared. Readiness for more sophisticated courses in mathematics at an earlier chronological age also was generally apparent. The supervisor inferred that a program of accelerated mathematics is feasible for academically talented pupils with no loss in proficiency.[17]

THE COLLEGE PREPARATORY SEQUENCE

The College Prepatory Sequence program consists usually of elementary algebra in grade 9, followed by 10th grade plane geometry supplemented by some solid and coordinate geometry, 11th-grade intermediate algebra, and a rather wide choice at grade 12.

---

[14]Schult, Veryl, Present Practices in Mathematics Instruction and Supervision, U. S. Dept. of Health, Education, and Welfare, Washington, D.C. 1966 p. 1.

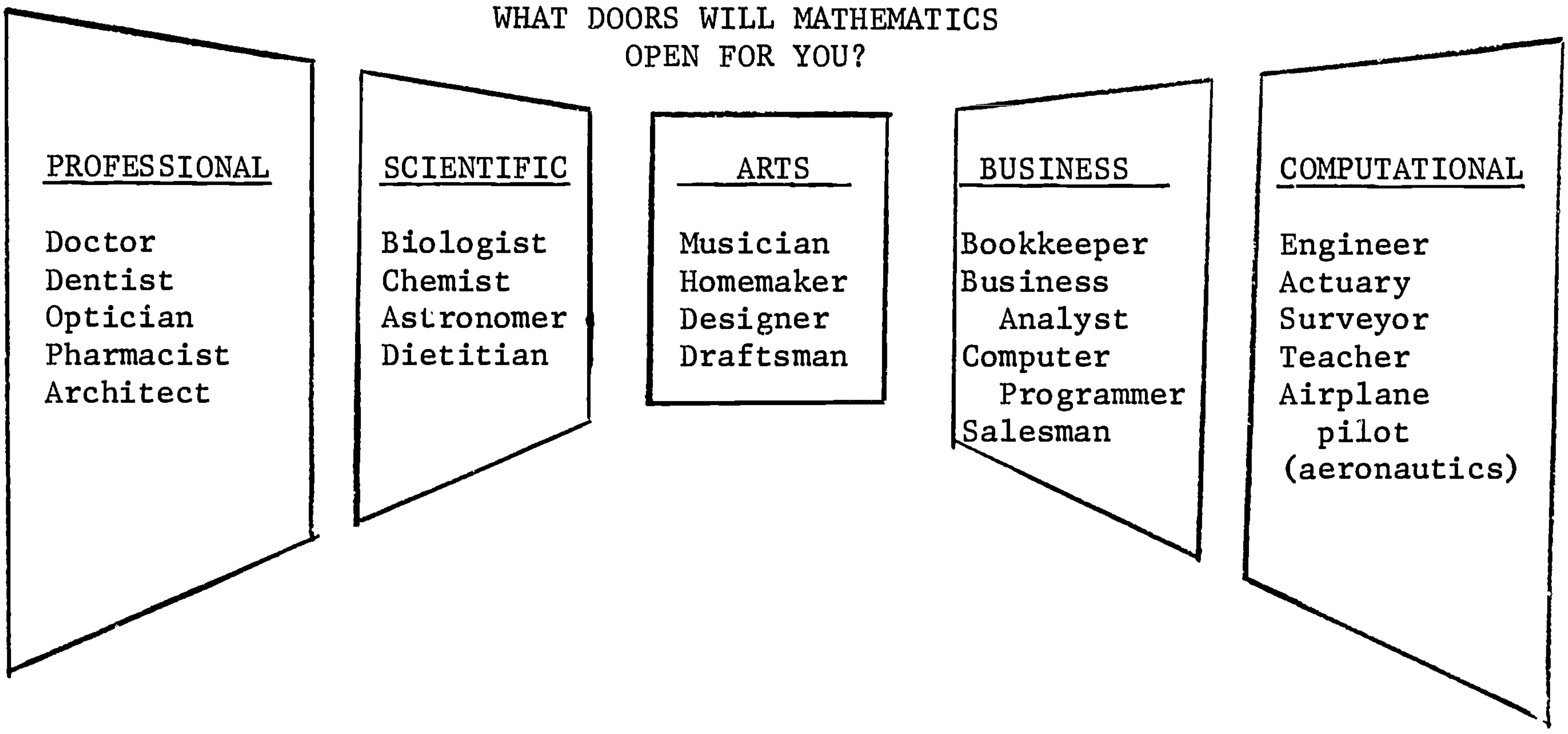
[15]Ibid., p. 1.

[16]Tolly, Harry R., A Follow-Up Study of Advanced Placement Student in Mathematics, Master's thesis, University of Nebraska, May, 1962.

[17]Ibid., pp. 2 and 3.

The present 12th grade college-preparatory courses are well described in a study by Lauren G. Woodby,[18] Emerging Twelfth-Grade Mathematics Programs, a recent publication of the U. S. Office of Education.

WHAT DOORS WILL MATHEMATICS OPEN FOR YOU?



For all of these vocations people now need mathematics, and many of these fields are using mathematics because of the new concepts. More mathematics is being taught.

A partial list of the articles found in the September, 1964, issue of the Scientific American[19] may also convince one of the importance of mathematics in many fields.

MATHEMATICS IN THE MODERN WORLD by Richard Courant
PROBABILITY by Mark Kac
MATHEMATICS IN THE PHYSICAL SCIENCES by Freeman J. Dyson
MATHEMATICS IN THE BIOLOGICAL SCIENCES by Edward F. Moore
CONTROL THEORY by Richard Bellman
COMPUTERS by Stanislaw M. Ulam

More mathematics will continue to be needed by more people. Permit me to give three examples of the use of mathematics in other disciplines on our own campus:

---

[18] Woodby, Lauren G., Emerging Twelfth Grade Mathematics Programs, Office of Education, U. S. Department of Health, Education, and Welfare, Washington, D. C. 1965.

[19] Op. Cit. Table of Contents.

1. Daily Nebraskan,[20] March 19, 1962--Former brace wearers may benefit from a new approach of orthodontics dentistry being started at the University.

   Dr. Sam Weinstein, Chairman of the Department of Graduate Orthodontics, and Donald Haack of the Department of Engineering Mechanics are responsible for what is now known as the theoretical mechanics approach to orthodontics.

   Shortly after Dr. Weinstein and Professor Haack began their research on moving teeth, the Universities of Indiana, Texas and California followed suit with similar programs built around a liason with engineers or physicists. Professors Haack and Weinstein have already received requests for information from throughout the United States and abroad.

2. Sunday Journal and Star,[21] December 5, 1965--A new dimension in medical and engineering education which could clear the way for a better understanding of the human heart is in formative stages at the University of Nebraska.

   The University, with a $655,000 grant from the National Heart Institute, has begun a five-year teaching program that will give selected students in medicine and biology, and cardiovascular scientists throughout the United States, an unusual opportunity for a year's study for engineering concepts and principles.

   The new teaching effort stated with the hope that theory and methods developed so highly by engineers in recent decades might be applied to research involving the heart and circulation.

   Under the program, carefully selected pre-and post-doctoral medical students, some of whom may qualify for annual living expense stipends of $3,000 to $6,000 will temporarily interrupt their medical training or research. They will engage in one year of studying at the University's Department of Electrical Engineering.

Richard Miles, a former University High School student, was one of the five students chosen for intensive instruction in the use and programming of digital and analog computers and studied such subjects as analytical geometry, calculus, probability and statistics, applied physics and electrical circuits. Students involved in cardiovascular research will be required to take more mathematics.

---

[20] "Dentist, Engineer Combine Talents", Daily Nebraskan, March 19, 1962.

[21] "N. U. Program Might Solve Mysteries of Heart", Lincoln Sunday Journal and Star, December 5, 1965.

3. Sunday Journal and Star,[22] January 24, 1965. Under ordinary circumstances, give some men a pair of dice and you have the beginnings of a chance game...Also known as aleatoric music (after the Latin word alea, or dice) some modern composers are using everything from computers to the patterns of tiny imperfections in paper to represent tones in obtaining sounds.

Superintendents usually like this kind of mathematics. But throw the dice to the University of Nebraska professors of music and before you know it, they will be composing "chance" music.

Will farm boys need the new mathematics? Titles of two more clipplings would indicate that the present as well as the future former will apply his knowledge of mathematics.

4. "Computer Predicts Corn Drying Results"[23]

5. "Future Hired Hand May Be Computer"[24]

We have heard much about the explosion of knowledge. Our century is also witnessing an "implosion" of knowledge. After centuries of increasing specializations, students of the various disciplines are beginning the difficult task of talking with each other again. It is no accident that the mathematician, Norbert Wiener, pioneered the inter-disciplinary science of cybernetics--the study of communication and control--, simply because mathematics is a universal language for any problem that can be defined precisely. The ability to discover, converse, and create in mathematical language is necessary for anyone who wants to take part in cybernetics.

The programs combining medicine and dentistry with engineering at the University of Nebraska which I mentioned earlier are similar inter-disciplinary attempts. But the young doctors who hope to capitalize on recent developments in engineering are, according to Dr. Lowenberg, not only studying applied physics and electrical circuits, but also digital and analog computers, analytical geometry, calculus, probability and statistics. The answers of the future necessitate a multi-disciplined approach. Dr. Weinstein explains that, "No single branch of basic science can be sufficent support for understanding all the problems involved."[25]

This is one more reason why it is important that many students learn not only to recite but to converse mathematically, not only to calculate but think mathematically--that they learn to speak precisely about all sorts of patterns which they discover for themselves and translate such discoveries into mathematics symbolism to be shared by others. Students need more knowledge about vectors, about probability, about matrices, about inequalities--these are the new concepts mentioned in Massie's study.

---

22 "They Bet Dice Roll in Harmony," Sunday Journal and Star, January 24, 1965.

23 "Computer Predicts Corn Drying Results," Summer Nebraskan, July 11, 1967.

24 Clipping from a Newspaper but unable to give date.

25 Op. Cit, Daily Nebraskan, March 19, 1962.

ANALYSIS OF PROGRAMS IN NEBRASKA AND NEIGHBORING STATES

Johnson's Study[26]

Let us take a look at what is happening in Nebraska at the ninth grade level. Evelyn Johnson completed an interesting study. (See Table 4)

TABLE 4.[27] RANK ORDER OF FREQUENCY OF USE OF VARIOUS TEXTBOOKS IN NINTH GRADE ALGEBRA CLASSES IN THE ACCREDITED PUBLIC SECONDARY SCHOOLS IN THE STATE OF NEBRASKA, 1965-66[a]

Rank	Title of the textbooks	Author(s)	Publisher	Copy-right	Number of schools using the textbooks
1	Modern Algebra, Structures and Methods	Mary P. Dolciani, Simon L. Berman, and Julius Frelich	Houghton Mifflin	1962	22
2	Algebra, Book I	A. M. Welchons, W. R. Krickenberger, and H. R. Pearson	Ginn & Co.	1962	19
3	Modern Algebra, Structures and Methods	Mary P. Dolciani, Simon L. Berman and Julius Frelich	Houghton Mifflin	1965	13
4	Algebra, Book I	A. M. Welchons, W. R. Krickenberger, and H. R. Pearson	Ginn & Co.	1960	10
5	New First Algebra	W. W. Hart, V. Schult, and J. D. Bristol	Heath & Co.	1962	9
6	Algebra, First Course	John Mayor and Marie Wilcox	Prentice-Hall	1961	9

[a] On the basis of responses from one hundred fifty-eight complete return schools.

---

[26] Johnson, Evelyn M., Survey and Analysis of the Ninth Grade Mathematics Program in the Accredited Secondary School in the State of Nebraska, 1965, 1966, Master's Thesis, University of Nebraska, Lincoln, May, 1966.

[27] Reproduced from Johnson's thesis, page 39.

Sixty-four percent of the schools are using texts written by only two different authors. More than one-half of the schools are using texts which contain "modern mathematics."

Urwiller's Study[28]

Stanley Urwiller of Grand Island, Nebraska, is carrying out an interesting study on spiraling assignments. In carrying out his study he contacted 395 schools in Iowa, Kansas, South Dakota and Nebraska. The Houghton Mifflin and the Ginn series are again used by approximately sixty percent of the schools. (See Table C)

TABLE C. TEXTBOOKS BEING USED IN SECOND YEAR ALGEBRA IN IOWA, KANSAS, SOUTH DAKOTA, AND NEBRASKA

Publisher	Authors	Number of Schools
Houghton Mifflin	Dolciani & Others	163
Ginn	Welchons, Krickenberger & Pearson	64
Merrill	Vannatta, Goodwin & Fawcett	29
Harcourt, Brace & World	Smith, Lankford & Payne	20
Holt, Rinehart & Winston	Keedy, Griswold & Schacht	15
Holt, Rinehard & Winston	Morgan & Paige	10
Laidlaw	Brown, Williams & Mongomery	9
SMSG		8
Singer	Mallory, Meserve & Skeen	7
American Book Co.	Shute & others	7
Heath	Fehr & others	7
Allyn & Bacon	Edgerton & Carpenter	6
Prentice-Hall	Mayor & Wilcox	6
Heath	Hart, Schult & Swain	6
American Book Co.	Kenner, Small & Williams	5
Van Nostrand	Peters & Schaaf	4
Ginn	Pearson & Allen	3
McGraw-Hill	Aiken, Henerson & Pingry	3
Heath	Hart, Schult & Bristol	3
Ginn	Weeks & Adkin	3
Holt, Rinehart & Winston	Nichols, Heimer & Garland	3
Others		14
TOTAL		395

[28] Urwiller, Stanley, The Effect of Spiraled Homework Assignments in Second Year Algebra and Student Attitudes Toward This Practice, A dissertation problem being carried out during the academic year 1967-68, University of Nebraska, Lincoln.

NEW DEVELOPMENTS

Goals for School Mathematics[29]

During the summer of 1963, a group of twenty-five professional mathematicians and users of mathematics, took time off from their normal work to review school mathematics from Kindergarten through grade 12 and to establish goals for mathematical education. Permit me to quote one paragraph from the report:

> The subject matter which we are proposing can be roughly described by saying that a student who has worked through the full thirteen years of mathematics in grades Kindergarten to 12 should have a level of training comparable to three years of top-level college training today; that is, we shall expect him to have the equivalent of two years of calculus and one semester each of modern algebra and probability theory...[30]

Might we expect the academic high school students to have the equivalent of three years of college mathematics at the time they graduate from high school? Is this a dream? During the fall of 1965 a series of conferences were held at Nova High School, Fort Lauderdale, Florida, to explore the feasibility of implementing a long-range curriculum development project for a non-graded, K-12 school based on the recommendation of the Cambridge Conference on School Mathematics. A cross-section of mathematicians, mathematics educators, and researchers attended these conferences. A report is now available with the proceedings, conclusions and recommendations reached at these conferences.[31]

It was this writer's privilege to speak with Burt Kaufman, Director of the project, at the annual meeting of the National Council of Teachers of Mathematics in Las Vegas in April, 1967. Kaufman informed me that he received a grant of $800,000 from the U.S. Office of Education and that the work for this project is now being carried on at the College of Education, Southern Illinois University, Carbondale.

The second paragraph in the "Overview" of the Proposal for Phase I of the project[32] explains briefly what is planned:

---

[29] Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics, Houghton Mifflin Company, Geneva, Illinois, 1963.

[30] Ibid, p. 7.

[31] Garret Foster, Burt A. Kaufman, Wm. M. Fitzgerald, A First Step Towards the Implementation of the Cambridge Mathematics Curriculum in a K-12 Ungraded School, Cooperative Research Project No. S-405, Florida State University, Tallahassee, Florida, 1966.

[32] Kaufman, Burt, Proposal for Phase I of a Project to Create an Individualized Mathematics Curriculum in the Spirit of the Cambridge Conference Recommendations; Comprehensive School Mathematics Project, University School, College of Education, Southern Illinois University, Carbondale, Illinois.

This is a proposal for a first phase in the development of an individualized mathematics program for students of ages 5-18, designed to provide individual students with top quality programs taking each student into mathematics as far as his ability and interest allow. The main vehicle for this instruction will be a series of activity packages using various learning media, constructed in cooperation with mathematicians and behavioral scientists.[33]

This program emphasizes individualized instruction, and each student has an opportunity to move as far and as fast as his desire and ability permit him.

Mathematical Games for Thinkers[34]

Another development is the use of mathematical games. Since space is a limiting factor only the names of a few games will be mentioned:

Equations: The Game of Creative Mathematics;[35] Real Numbers; On-Sets: The Game of Set Theory; WFF'N PROOF: The Game of Modern Logic. WFF'N PROOF provides practice in abstract thinking and an opportunity to learn some mathematical logic.

An interesting article[36] entitled, "Games in the Classroom", may be found in the April, 1967, issue of the Saturday Review.

Low Achievers in Mathematics

The very fact that jobs under automation demand higher skills creates certain problems. Since the need for unskilled labor will decline and the need for skilled and semi-skilled technicians will grow, automation brings with it the possibility of a distinct upgrading of jobs. "The hand trucker of today replaced by a conveyor belt might become tomorrow's electronic engineer," as one industrialist has put it.

The spread of automation, however, will call for training and retraining on a much broader scale. "Many of today's electricians will have to learn electronics if they are to retain their skilled status," according to the National Manpower Council. They state that "Pipefitters may have to learn hydraulics. A skilled worker who formerly measured with calipers and now uses a micrometer will soon have to learn to work with tolerances measured with light waves...there may be almost no place left for the unskilled worker." Mathematics has an important part to play with this training and retraining.

In the years ahead, more mathematics, not less, must be taught to more students. The high school needs to provide sound mathematical training for our future leaders of science, mathematics, and other learned fields. Few persons would question this, or the fact that the average person needs an

[33] Ibid., Page 4.

[34] These games were displayed.

[35] The games mentioned may be purchased from WFF'N PROOF, Box 71, New Haven, Connecticut, 06501.

[36] Carlson, Elliot, "Games in the Classroom", Saturday Review, April, 1967.

understanding of the fundamental processes of arithmetic. But how about the low achiever in mathematics? Do they not require a mathematics curriculum geared to their needs and demonstrated learning ability? We must offer programs for those of low ability, for the knowledge of mathematics is essential for each and every student if he is to function effectively in today's world.

> That the high school dropout is a likely candidate for the growing proletariat class in the United States is becoming an insistent contemporary theme. It is anticipated that eight million youths in the United States will terminate their education in the present decade without obtaining a high school diploma. "Dropout: The New Lost Generation" is the way Michael Harrington characterizes them. Secretary of Labor, W. Willard Wirtz, is more severe in his summary, when he terms them: "The human slag heap" and states that "Each dropout will cost taxpayers $1000 per year for the rest of his life. The problem of the dropout," continues Wirtz, "is destined to be one of the greatest that our Republic has faced."[37]

Teaching mathematics to the low achiever is a difficult task. Because of this difficulty some fifty educators including mathematicians, sociologists, psychologists, high school teachers, college professors, mathematics consultants, school administrators, and representatives from business and industry met on April 21 and 22, 1966, at Central College in Pella, Iowa, and in Des Moines, Iowa, to formulate plans for actively attacking the problem. The purpose of the Conference was to enlist the cooperation of schools and educators from throughout the United States, in an "action" project.

<u>Current Programs</u>

Los Angeles City Schools

The Los Angeles City School districts are engaged in various activities which relate directly or indirectly to the problems of the low achiever. A mathematics laboratory kit is used with slow learners. You may be interested in this experimental kit.

> Classes in twenty schools are presently experimenting with mathematics kits of inexpensive items which were distributed on the basis of one kit per student. The kit contains materials such as rulers with graduations of half inches or quarter inches, a tape measure, compass, number cards, abacus and a simple slide rule. The teaching suggestions which accompany the kits present ways of using each item. We think we have developed some unusual ways of using these materials. Incidentally, it is important to set the stage for using some of these items. For example, a tape measure may be acceptable, at first, to a class including teen-age boys, but if its initial use is to measure muscle-flex of various members of the class and make a chart of the data, then the tape becomes acceptable. After that it can be used to measure the accuracy of estimates of length. Having established the tape as being respectable, the teacher can use it to advantage in teaching facts about fractions.

---

[37] Rosenbloom, Paul C. Dr., Director, <u>Project Conference Report</u>, p. 7. The comments were made by Dr. Lucious F. Cervantes, S. J., in a talk entitled "The Dropouts and Mathematical Achievement."

Then, too, we are experimenting with the use of computers in the classroom. This may sound "far out" to some people. Fortunately for us, at least two companies have developed desk size computers which cost less than $5,000 each. Teachers learn, and later teach, some uses of these computers. Students learn to program simple processes. To do this and find that one can get correct answers is a thrilling experience. I do not mean that we recommend that these students attempt difficult solutions, but the machines provide a new approach to known processes. For example, in order to devise a program for division, the student must recognize the process as repeated subtraction. He may, for the first time, understand what he is doing as her performs division.[38]

The Iowa Program[39]

The Iowa State Department of Public Instruction has developed a program in mathematics instruction to catch and hold the interest of the student who has gotten behind his classmates, lost hope and is likely to drop out of school. The program was developed by teachers and administrators in classroom experiments over a two-year period in the spirit of the 1964 conference on low achievers in mathematics sponsored jointly by the U.S. Office of Education and the National Council of Teacher of Mathematics. During the past year several hundred schools have incorporated many of the techniques developed by the Iowa program into their remedial mathematics programs. The five basic techniques of the Iowa program are:

1. Unit-per-day approach. Change of pace to keep students interested.
2. Real-life case history mathematics problems on local letterheads of local companies.
3. Mathematics laboratory with many multi-sensory and audio-visual aids.
4. Flow charting--the link between the abstract and the concrete.
5. Involvement discovery experiments.

Sarah Greenholz, Supervisor of Mathematics of the Cincinnati Public Schools, speaking at the National Council of Teachers of Mathematics Convention in New York referred to the Iowa approach as the most significant breakthrough in reaching low achievers.

Oakland County Mathematics Project[40]

The Oakland Schools near Pontiac, Michigan, have a total enrollment of approximately 250,000 students enrolled in grades one through twelve. David Wells, Director of Instruction and Mathematics Education for Oakland Schools, is directing the Oakland County Mathematics Project. Wells and his staff are planning to prepare a four-year high school mathematics program for the non-college aspiring students. The Oakland Schools have received a grant from the U.S. Office of Education totaling $660,000.

---

[38] Rosenbloom, Ibid., pp. 33-35.

[39] Groenendyk, Eldert A., Mathematics Consultant, Iowa State Department of Public Instruction, Iowa Program.

[40] Oakland County Mathematics Project, Final Report to the U.S. Office of Education on Project #1860, Title III - Public Law 89-10, November, 1967.

Overview of the Proposed Oakland County Project

The Oakland County Project which is being proposed is a continuation and extension of the Oakland County Mathematics Project, which has been proceeding under an E.S.E.A. Title III planning grant. Under the planning grant, considerable information has been collected about mathematical requirements of jobs and useful materials designed for non-college aspiring students. Content outlines for a four-year sequence of courses for these students have been formulated.

The proposed operational grant would involve the following activities, building upon the work done under the planning grant: (1) Writing and preparing materials based on the content outlines developed. These materials will incorporate an inductive approach featuring laboratory activities, mathematical games, simulation, and a heavy emphasis on critical thinking. Provision for individualized instruction will be incorporated. (2) Field-testing the materials in pilot classes, and revising the materials on the basis of the field-testing. (3) Providing inservice training for teachers. This training will emphasize effective techniques of teaching mathematics to non-college aspiring students. (4) The preparation of effective teaching aids to supplement the written materials. (5) The careful evaluation of the materials themselves, and their effect on students. This will include evaluating the amount of mathematics learned, the attitude of the student toward mathematics and toward schools, and the degree to which critical thinking has been improved.[41]

The reader's attention especially is called to the use of materials which incorporate an inductive approach featuring laboratory activities, mathematical games, simulation, and a heavy emphasis on critical thinking. Another innovation is providing for individualized instruction. Another unique feature will be the preparation of effective teaching aids to supplement the written materials.

Other Projects Dealing with Non-College Aspiring Students

The Des Moines, Iowa, Public Schools have developed a project known as LAMP (Low Achiever Motivational Project). The laboratory approach is used extensively. The special features of LAMP are: (1) use of local business problems; (2) flow charting; (3) individual files which give a day by day report of the student's progress; (4) Enrichment-Student-Project; (5) student reports in class (these are extra-credit); (6) use of mathematical games.[42]

The Wynne, Arkansas, Public Schools developed an extensive set of materials. These were prepared under the direction of Gene Catterton and funded by a Title III (ESEA) grant. The striking features are: (1) the use of flow charts (similar to flow charts for computer programs) as models to follow in using algorithms; (2) organization of the materials on the basis of self-contained one-day lessons (some sequence is imposed on this by grouping the material into chapters); (3) the use of real-life problems obtained from local businesses and industries and printed with their letterheads; (4) material designed for use with printing calculators.[43]

---

[41] *Ibid.*, pp. 101-102.

[42] Zimmerman, Joseph T., LAMP (Low Achiever Motivational Project). Des Moines, Iowa; 1966.

[43] Catterton, Gene, and others, Drop-In Mathematics. Wynne, Arkansas: Wynne Public Schools, Summer, 1966.

The Minnesota Department of Education has developed a source-book entitled Applied Mathematics. This publication is organized into the following topic areas: Computing and Computing Machines; Probability, Risk, and Insurance; The Language of Science; Facts from Figures and Figuring From Facts (statistics); Fact or Fancy (logic); Making a Million or Something less (investment); Stretching Your Dollar; Financing Freedom (taxation); Money and Your Master; Blast Off to the Future (coordinate systems, the conic sections, space travel).[44]

Information relative to other projects could be given. The best source for a collection of information about projects dealing with non-college aspiring students, and collecting for use with these students, along with recent practices and experimentation in general mathematics, is the Final Report to the U.S. Office of Education on Project Number 1860, Title III, Public Law 89-10, produced by the Oakland County Mathematics Project.[45]

Guidelines For Administrative Provision For The Low Achiever In Mathematics

The school administrator is a most important factor in the success or failure of a program for the low achiever in mathematics. He must initiate and provide for an effective orientation program for the staff and the community; he must see to it that teachers of low achievers are given proper recognition and special acknowledgement for the importance of their work. He has the job of interpreting the program to the community. He bears the responsibility of drawing leaders in the community in business, commerce, and industry into the development of a program that will prepare the low achievers for a productive role in the community.

Below are the guidelines for the administrators:[46]

1. The administrator should provide mathematics courses for each year (K-12) for the low achiever.

2. The administrator should provide for maximum individual growth of low achievers by careful grouping.

3. The administrator should see that order is maintained in the classrooms of low achievers.

4. The administrator should provide semiprofessional aides to the teacher of the low achiever.

5. Citizens, community leaders, and representatives of business, commerce and industry should be involved in the development of a mathematics program for low achievers.

---

[44] Applied Mathematics. Curriculum Bulletin No. 20A. St. Paul, Minnesota: State of Minnesota Department of Education, 1964.

[45] Op. Cit., Oakland Schools, An Intermediate District of School Administration, Campus Drive, Oakland County Service Center, Pontiac, Michigan 48053.

[46] Woodby, Lauren G., The Low Achiever in Mathematics: OE-29061, U.S. Department of Health, Education and Welfare, Office of Education Bulletin 1965, No. 31., pp. 85-88.

6. The school administrator must involve the parents of the low achievers and establish lines of communication between the parents and the school.

7. Pupil's marks should tell the truth.

8. The administrator must select teachers who are competent in mathematics and qualified by both preparation and temperament to teach low achievers.

9. Intensive professional inservice education programs must be provided for teachers of low achievers in mathematics.

10. Special guidance is needed for special pupils.

11. The administrator must provide opportunities for teachers to engage in research and experimentation.

BECKMANN'S STUDIES--ARE STUDENTS ACHIEVING GREATER COMPETENCY IN MATHEMATICS TODAY THAN SEVENTEEN YEARS AGO[47]

Introduction

Between 1950 and 1965 there was a great deal of activity in the revision of mathematics programs in the elementary and high schools of this country. Most elementary and secondary teachers, principals, administrators, supervisors, and mathematicians have heard about the new experimental programs in mathematics--the School Mathematics Study Group, the University of Illinois Arithmetic Project, The Maryland Study, the Boston College Program, the Madison Project, and others, which have influenced textbook writers. Public school and college personnel and parents are eager to know whether students are more mathematically literate now than they were before they were exposed to the "modern mathematics." Many of these same persons are more concerned that the student know the basic mathematics needed for personal use by everyone--not just by those who plan to go to college or engage in technical professions and trades--than that the student be familiar with the new terminology, precise definitions, units on sets, vectors, topology modular arithmetic and the like.

In 1944, the board of directors of the National Council of Teachers of Mathematics created a Commission of Post-War Plans. Its assignment was to decide the basic mathematics needed for personal use by the post-war citizen and to prescribe realistic goals in the post-war teaching of mathematics on the basis of this decision.

---

[47] Beckmann, Milton W., Mathematical Competency and Relative Gains in Competency of Pupils in Algebra and General Mathematics, Doctor's Dissertation, University of Nebraska, Lincoln, 1951.
An article was recently submitted to the Editor of The Arithmetic Teacher published by the National Council of Teachers of Mathematics. It is a study similar to the one made in 1951.

Thesis Number 1, headlining the Commission's various reports, was mathematical literacy for all who can possibly achieve it.[48] All citizens, as well as teachers of mathematics, should be vitally concerned with the first thesis.

Other recommendations of the Commission follow: the teaching of arithmetic can be and should be improved; we must give more emphasis and much more careful attention to the development of meanings; we must abandon the idea that arithmetic can be taught incidentally or informally; the large high school should provide a double track in mathematics; we should differentiate on the basis of needs; the sequential courses in high school should be greatly improved; we must learn to administer drill (repetitive practice) much more wisely; the mathematics for grades 7 and 8 should be planned as a unified program and should be built around a few broad categories; the mathematics program of grades 7 and 8 should be so organized as to enable the pupils to achieve mathematical maturity and power; and so forth. But this primary aim -- mathematical literacy for all -- was regarded throughout as the most important job facing the Commission.

Mathematical Literacy Defined

How much mathematics is needed by every citizen? How much mathematics does it take to be "mathematically literate" -- to fit Mr. Average Citizen culturally and vocationally to his modern, scientifically changed environment? The essentials for functional competence in mathematics are placed as questions in a check list by the Post-War Commission.[49] If the 29 questions can be answered with a "yes," the student can feel somewhat secure when it comes to dealing with the problems of everyday affairs.

Competence in handling this kit of tools is very useful in solving the great majority of the average man's problems. It is the basic mathematics needed in a great many vocations. We realize that the list is relatively short and for some absurdly simple. Nevertheless, most employers who hire high school graduates would be greatly pleased if they were certain that the essentials for functional competence in mathematics were achieved by all students who worked for them. This check list of 29 competencies suggests that competence in mathematics is almost as crucial as literacy in communication; therefore, the school must guarantee competence in these specific matters to all students who can possibly achieve it.

During the school years 1950-1961, the writer designed a study[50] to measure the level of mathematical competency as defined by the 29 competencies promulgated by the Commission on Post-War Plans, as a result of the study of mathematics through the first eight grades. The level of mathematical attainment was astonishingly low.

One cannot help but speculate what the situation is like today when functional competence in mathematics is still essential. A great deal of money and effort the past fifteen years has gone into experimentation to improve the

---

[48] Mathematical literacy is defined under "Mathematical Literacy Defined."

[49] Schorling, Raleigh, (Chairman), "The Second Report of the Commission on Post War Plans," The Mathematics Teacher, 38: 197-198, (May, 1945).

[50] Ibid.

teaching and content of mathematics at the elementary level. During the school year 1965-1966 the writer again designed a study to measure the level of mathematical competency achieved as a result of the study of mathematics through the first eight grades with approximately the same number of students in approximately the same schools tested during the school year 1950-1951. Will the studies show that students today are as mathematically competent as students were 15 years ago? Is the level of mathematical competency as defined by the 29 competencies found to be higher or lower for students completing the eighth grade today than it was 15 years ago?

Statement of the Problem

Beckmann's study was designed to measure and compare the mathematical literacy for students completing the first eight grades during the school years 1950-1951 and 1965-1966.

Organization and Design of the Study

The tests were given to both groups during the fall; consequently, the students were just beginning work in the ninth grade. Forty-two Nebraska high schools, with 1,296 students were selected for final inclusion in the 1951 study. In choosing the forty-two schools, consideration was given to geographical location so that all areas of Nebraska were represented. For the purpose of comparison it was necessary to include the same schools. Forty Nebraska high schools with 1,377 students were selected for final inclusion in the 1965 study. One school no longer existed because of school reorganization. The other school consented to participate, but failed to carry out the necessary instructions.

Evaluative Instrument

In planning the first study, it was decided to use a mathematics test which would measure functional competence in mathematics in terms of the 29 competencies found in the check list. Since no satisfactory measuring instrument could be found, it was decided that one should be constructed. A detailed explanation of the construction of the Mathematical Literacy Test is given in Chapter V of a study conducted by the writer.[51] The steps taken in the construction of the test followed rather closely the usual procedure in test construction. The same Mathematical Literacy Test was used again in 1965.

Findings

The Mathematical Literacy Test is made up of 109 items. The initial mean score on the Mathematical Literacy Test for 1,296 students entering the ninth grade in 1951 was 45.66. The initial mean score on the same test for 1,377 students entering ninth grade in 1965 was 55.02. This is a difference in means of 9.36 which yields a t score of 4.5 which is significant beyond the 0.001 level.

We can say that students entering the ninth grade in Nebraska schools are much more mathematically literate today than they were 15 years ago.

---

[51] Ibid.

It is difficult to pinpoint the reasons for this improvement. One might make the following assumptions: the students were exposed to modern mathematics; teachers are better prepared mathematically; the total academic atmosphere in the schools has improved. You might think of some other reasons for the appreciable gains in students' mathematical literacy.

IMPLEMENTING NEW DEVELOPMENTS AND NEW TECHNIQUES

Qualifications of the Teacher

The teacher of mathematics should have a wide background in the subjects he will be called upon to teach. It will not be possible to accomplish this program completely with most prospective teachers during their undergraduate training. However, enough training may be given to enable the teacher to get well on the way to complete accomplishment.

A teacher cannot aid intelligently in reorganizing the high school curriculum in mathematics without a reserve of mathematical knowledge. Every school should provide one or more teachers with a Master's degree in mathematics. The Cambridge Report stressed that a competent high school student in mathematics be given an opportunity to have a level of training comparable to three years of top-level college training. Kaufman's program at Carbondale, Illinois, which was discussed, is implementing the Cambridge Report. Very few small schools would employ teachers competent enough in mathematics to handle such quality programs.

The College Entrance Examination Board offers the Advanced Placement Program in the interest of able students, secondary schools which enable these students to take work commensurate with their abilities, and colleges which welcome freshmen who are ready for advanced courses. A student seeking advanced placement in college mathematics may have to pursue a somewhat different secondary school program from that traditionally offered in most schools. A course for the eleventh year would consist of a continuation of algebra, analytical geometry, and trigonometry and thus providing a model introduction to analysis. A twelfth grade course would consist of a substantial introduction to the calculus, supplemented by certain selected topics in algebra and analytic geometry. The Commission of Mathematics of the College Entrance Board feels that only a small percent of the mathematics teachers could teach the twelfth grade course. This would imply that the larger schools would offer accelerated students this opportunity. Seldom would a student in a smaller school be given this opportunity. Some measure of how far we are from teaching what children can learn is indicated by a few simple facts. We are the only industrialized country in the world which does not teach calculus to the top 10 or 15 percent of the 17-year olds.

Approximately fifty percent of the students now entering the University of Nebraska register for Mathematics 114--Analytics and Calculus.

Many colleges do not offer college credit for students taking algebra and trigonometry while in college. This includes the Engineering College at the University of Nebraska.

A teacher with only a baccalaureate degree would have difficulty teaching the accelerated learner. One needs also to have special preparation for the teaching of the low achiever in mathematics.

One or more teachers on a staff should be trained in using techniques of teaching mathematics to non-college aspiring students. If better teaching is to help the low achiever in mathematics, then better teachers are needed. By "better teachers" we mean better teachers prepared psychologically. Bad teaching or writing may make a bigger difference for slow learners than for average students.

No one knows much about the low achiever's capacity to learn, but the limits may be far beyond what he now learns. If these pupils do not acquire abilities which make them employable, they become lifelong public charges. In most schools, the teachers of slow learners are either the lowest in the pecking order in their respective departments or specialists on low ability students with little special knowledge of mathematics. An effective curriculum for slow learners in every school will have to be developed by the few highly imaginative mathematics teachers that can be recruited.

Several mathematics teachers in each system should be alert to new developments with the mathematics laboratory. Programs of mathematics in the elementary and secondary schools require libraries of materials. The elementary teacher with initiative will have available a wealth of instructional aids to help children as they discover ideas of mathematics and learn to think with these ideas. The secondary teachers should learn the use of the transit, sextant, slide rule, computers and many other devices. One also needs to be informed of the new mathematical games.

Provision for Varied Abilities

The high school needs to come to grips with its dual responsibility, namely, (1) to provide sound mathematical training for our future leaders of science, mathematics, and other learned fields, and (2) to insure mathematical competence for the ordinary affairs of life to the extent that this can be done for all citizens as a part of a general education appropriate for the major fraction of the high population. To accommodate these students of varied abilities the high school of 500 or more pupils should provide at least two tracks in mathematics, and in some cases three and four tracks. The sequential courses should be reserved for those pupils who, having the requisite ability, desire, or need such work. New and better courses should be provided in the high schools for a large fraction of the school's population whose mathematical needs are not well met in the regular sequential courses.

Mathematics Supervisor

An increase of 650 percent in the number of state supervisors of mathematics?[52] Incredible! Yet that is what happened in the period 1955-65. Comparable to the increase in the number of State mathematics supervisors is the increase in the number of mathematics supervisors in school systems within the states. It is unfortunate that the small school does not have the advantage of employing a mathematics supervisor.

---

[52] Schult, Veryl, Op. Cit, pp. VII, 38, 39.

The responsibilities of mathematics supervisors have greatly increased due to expanding curriculums, larger numbers of inexperienced teachers, new teaching and learning aids, greater variety of textbooks, new methods of teaching, and a greater need to interpret programs to the public. A few of the typical activities of mathematics supervisors in large city school systems are described below:

- One or two-day workshops during the school term
- Departmental meetings on current topics and problems
- Consultants meeting with special groups
- Courses financed through NDEA
- Courses arranged by the mathematics department in cooperation with a nearby college
- Classes in electronic computation

Each state now has one or more supervisors of mathematics in setting up strong state programs and coordinating them, and to assist in carrying out the provisions of the National Defense Education Act (NDEA). Also many of the larger local schools have employed supervisors to coordinate mathematics from kindergarten through grade 12.

A LIST OF CHARACTERISTICS THAT DISTINGUISH THE MODERN SECONDARY SCHOOL MATHEMATICS CURRICULUM FROM THE TRADITIONAL MATHEMATICS CURRICULUM

Characteristics Common to Modern Mathematics Courses at All Levels

1. Structure emphasized. Building on the foundation laid in elementary school, the secondary school curriculum emphasizes structure in mathematics. In algebra and analysis courses this emphasis takes the form of broadening and deepening students' understanding of the properties of the real number field and the application of these properties to the justification of transformations and manipulations.

2. Careless use of language avoided. Careless use of the specialized vocabulary and symbols of mathematics is avoided. A few new symbols are introduced when needed to make clear mathematical concepts, and some definitions are re-worded to avoid confusion over meaning, but the emphasis is on using in an unambiguous manner the symbols and terms already familiar to the student.

3. Understanding emphasized. There is renewed emphasis on helping the student understand what he is doing when he performs the tasks required of him in a mathematics course. This goal is approached through a variety of means, such as application of both inductive and deductive reasoning to the development of mathematical generalizations and a noticeable reliance on directed discovery teaching, particularly in junior high school courses.

4. Set language used. The language of sets is used in clarifying and simplifying definitions and important concepts.

Characteristics Common to Courses in Grades 7 and 8

1. Work with other numeration systems continued. Non-decimal numeration systems are used to give meaning to the algorithms for the four fundamental operations of arithmetic, to introduce exponential notation, to introduce the idea of a polynomial as well as to point out the advantages of place-value notation and the importance of the symbol for zero.

2. Verbal problems revised. Problems dealing with insurance, taxes, banking, and other concerns of the adult world are grently de-emphasized or eliminated altogether. An attempt is made to substitute problems of more immediate interest to the student.

Characteristics Common to Courses in Beginning Algebra

1. Deductive reasoning applied. Deductive reasoning is applied in the development of some algebraic generalizations. Students are provided with ample illustrations of the fact that deductive proof need not be confined to geometry.

2. Function concept introduced. The function concept is introduced and developed in an orderly, meaningful manner. Sets of ordered pairs are commonly used in formulating the definition of this fundamental mathematical concept.

3. Inequalities treated along with equations. Students are given opportunities to solve and to graph inequalities as well as equations.

Characteristics Common to Courses in Geometry

1. Coordinate geometry introduced. Coordinate geometry is introduced and used in proving theorems and in solving problems.

2. Pattern of logical discourse emphasized. More attention is paid to the pattern of a logical discourse than is usual in traditional geometry courses. An attempt is made to help the student understand the parts played by undefined terms, definitions, postulates, and theorems in the pattern of postulational thinking.

3. Modified postulate set used. Geometry is based upon a set of postulates that is essentially the same as that used in traditional geometry courses with some modifications and additions to eliminate logical inadequacies and permit a sounder treatment of the contents of this course.

4. Plane and solid geometry integrated. Solid geometry is eliminated as a separate course, and important concepts from solid geometry are taught along with the analogous material in plane geometry.

Characteristics Common to Courses in Grades 11 and 12

1. Emphasis in trigonometry changed. In trigonometry courses emphasis is placed on the trigonometric functions as functions of real numbers and on trigonometry's relation to vectors and complex numbers. Triangle solving and surveying and navigational problems do not dominate the course.

2. New topics introduced. Number fields, vectors, matrices, probability and statistics, and other topics new to the secondary school curriculum are introduced, and a more formal and advanced treatment is given to such fundamental mathematical concepts as variable, relation, and function.

BIBLIOGRAPHY

Applied Mathematics. Curriculum Bulletin No. 20 A. St. Paul, Minnesota: State of Minnesota Department of Education, 1964.

Beckmann, Milton W., Mathematical Competency and Relative Gains in Compentency of Pupils in Algebra and General Mathematics, Doctor's Dissertation, University of Nebraska, Lincoln, 1951.

Begle, E. G., Director of School Mathematics Study Group, School of Education, Stanford University, Stanford, California 94305.

Carlson, Elliot, "Games in the Classroom", Saturday Review, April, 1967.

Catterton, Gene and others, Drop-In Mathematics, Wynne, Arkansas, Wynne Public Schools, Summer, 1966.

"Computer Predicts Corn Drying Results" Summer Nebraskan, July 11, 1967.

Deans, Edwina, Elementary School Mathematics, New Directions, U. S. Department of Health, Education and Welfare, Washington, D. C. 2963.

"Dentists, Engineers, Combine Talents" Daily Nebraskan, March 19, 1967.

Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics, Houghton Mifflin Co., Geneva, Illinois, 1963.

Hawthorne, Frank S., Chief, Bureau of Mathematics Education, New York State Education Department. Elementary School Mathematics. New York State School Boards Association, Inc. Vol 8, No. 6, December, 1966.

Foster, Garret R., Burt A. Kaufman, W. M. Fitzgerald, A First Step Towards the Implementation of the Cambridge Mathematics Curriculum in a K-12 Ungraded School, Cooperative Research Project No. S-405, Florida State University, Tallahassee, Florida, 1966.

Groenendyk, Eldert A., Mathematics Consultant, The Iowa Program, Iowa State Department of Education, Public Instruction.

Johnson, Evelyn M., Survey and Analysis of the Ninth Grade Mathematics Program in the Accredited Secondary School in the State of Nebraska, 1965, 1966, Master's Study, University of Nebraska, Lincoln, 1966.

Kaufman, Burt, Proposal for Phase I of a Project to Create an Individualized Mathematics Curriculum in the Spirit of the Cambridge Conference Recommendation: Comprehensive School Mathematics Project, University School, College of Education, Southern Illinois University, Carbondale, Illinois.

"N. U. Program Might Solve Mysteries of Heart", Lincoln Sunday Journal and Star, December 5, 1965.

Massie, Ronald O., The Construction and Use of a Test to Evaluate Teachers Preparation in Modern Mathematics, Doctor's Dissertation, University of Nebraska, Lincoln, 1967.

Oakland County Mathematics Project, Final Report to the U. S. Office of Education on Project #1860, Title II Public Law 89-10, November, 1967.

Rosenbloom, Paul C., Director, Project Conference Report, The comments were made by Dr. Lucious F. Cervantes, S. J. in a talk entitled, "The Dropouts and Mathematical Achievement."

Schorling, Raleigh, Commission on Post-War Plans, "Second Report of the Commission on Post-War Plans", The Mathematics Teacher, Vol. 37, May, 1945.

Schult, Veryl, Present Practices in Mathematics Instruction and Supervision, U. S. Department of Health, Education and Welfare, Washington, D.C. 1966.

"They Bet Dice Roll in Harmony," Star, January 24, 1965.

Tolly, Harry R., A Follow Up Study of Advanced Placement Students in Mathematics, Masters thesis, University of Nebraska, May, 1962.

Urwiiler, Stanley, The Effect of Spiraled Homework Assignments in Second Year Algebra and Student Attitudes Toward This Practice, A dissertation problem being carried out during the academic year 1967-68, University of Nebraska, Lincoln.

Woodby, Lauren G., Emerging Twelfth Grade Mathematics Programs, Office of Education, U.S. Department of Health, Education and Welfare, Washington, D. C. 1965.

Zimmerman, Joseph T., LAMP (Low Achiever Motivation Project). Des Moines, Iowa.